Solving Zeno's Paradoxes

I will be discussing solutions to a paradox usually attributed to the ancient Greek Philosopher Zeno. Zeno posed a paradox that involved a runner trying to reach the finish line in a race. In order to reach the finish line the runner must first reach the point between the start and the finish. Before he can reach that point he must reach the quarter point. The series continues in this fashion until it seems that our runner must complete an infinite amount of journeys to begin movement, much less to complete the race. Therefore Zeno suggests that the runner can never reach the finish line because an infinite amount of tasks cannot be completed in a finite amount of time. Modern-day philosopher Max Black proposes the solution that the runner does not indeed have an infinite number of journeys to make. Black argues that the there is just one journey from the start to the finish.

Before we can continue we must be clear on the definitions of some terms. The word journey as used in Zeno's paradox is taken to be any given distance that the runner must traverse. For the runner to move from one point to another is to complete a journey. He does not mean undertaking a journey off to some distant land. Black will use the concept of some machines that perform infinite numbers of tasks to prove his argument. When he speaks of the number of tasks that his machine must perform he simply means that the machine has taken the steps necessary to accomplish its goal. An infinite series is a series of summations that goes on without end. If the series converges then the sums of its finite segments get closer to some value as more segments are taken. Eventually it is close enough that mathematicians are satisfied to say that it equals that value. The final definition is that of infinity. In essence, to say that something goes on for infinity means that it will never end. As a consequence of our runner having an infinite amount of journeys to make he can never reach the end. He will always have another journey to make.

Max Black's first step in solving Zeno's paradox is to say that it is impossible to complete an infinite amount of tasks in a finite amount of time. That is, he agrees with Zeno's premise. Black illustrates his point by having us envision some infinity machines. These are machines that are able to complete infinite tasks. He begins by having a machine that has an infinitely long tray filled with an infinite amount of marbles. This machine will have completed its task if all of the marbles are moved onto another tray. Black's first problem with such a machine is the inability to verify that there is indeed an infinite amount of marbles in the first tray. Realizing that other philosophers could possible conceive of a solution to not knowing how many marbles are in the first tray, Max decides to refine his machine. He now has us picture another machine. We will also reduce the number of marbles to be moved to one. This machine will continuously move the marble from one side to the other an infinite amount of times. Some device on the machine always returns the marble back to the original tray as soon as the machine puts it on the final tray. Each time it picks the marbles up with greater and greater speed. At the end of four minutes it comes to a stop, since it's moved the marble an infinite amout of times. At the end of four minutes the machine has not completed its task. No sooner had it put the marble down as it was returned to the original spot. Therefore, this machine cannot complete its task. To have completed its task the marble would have to be in the second tray. At the end of the infinite movements, however, the ball has been returned back to the beginning. Therefore, no matter how fast it runs, the machine cannot complete its task.

Not yet satisfied that his solution is foolproof, Black introduced another machine. This third machine works in concert with the second machine. Its actions are exactly opposite that of the second one. When the second machine puts the marble on the right, the third one puts it on the left side. At the end of the four minutes if the second machine has completed its task, the marble is on the right. If the third machine has completed its task, the marble is on the left. However, since our one marble cannot be in two places at once, neither machine has completed its task.

Now we are given a second pair of machines. These machines handle two marbles, one on each tray. Without pausing they proceed to move the marble that they have on their side to the other side. Then they go and move the marble that is now on their side. As one can reason, there will always be a marble on each tray. As one machine moves a marble to the left another one moves it to the right. They can run forever (although they only run for four minutes) and neither machine will ever have completed its task. He also explains that a machine that handled smaller and smaller marbles would not solve the problem either. It would still have to complete an impossible task. Since none of his machines can complete their infinite tasks, Max Black says that an infinite amount of tasks cannot be performed in a finite amount of time.

In order to criticize Black's points, opponents might draw analogies to Thomson's lamp. Thomson's lamp also set out to show that an infinite amount of things could not be done in a finite amount of time. Thomson's lamp consisted of a lamp that was switched on and off continuously an infinite amount of times. Each time that the switch was flipped, the time interval would be halved. After four minutes Thomson asked whether the light was on or off. The lamp cannot be on or off because each time you turned it on during the infinite series you turned it off. Since a lamp cannot be in a state of neither being on nor off, an infinite amount of things cannot be completed in a finite amount of time because infinite series leaves us in a state of contradiction. Sainsbury, however, contends that the final state of the lamp has nothing to do with the infinite series since it is not part of the infinite series. As he describes it, the series generates new states of being off and on within the series. Each switching is twice as fast as the previous. However, this only describes points within the series. We are asked about the lamp after the series. The state of the lamp after the series was not generated by the series, and, therefore, has nothing to do with the fact that it is switched on and off an infinite amount of times. Therefore, according to Sainsbury, Thomson's lamp does not prove that it is impossible to do an infinite amount of things in a finite amount of time. Critics might compare the infinity machines to Thomson's lamp and claim that the outcome has nothing to do with the series. However, unlike the lamp, which is in an impossible state at the end, Black's machines are not in a state of physical contradiction when the infinite movements have ended. The ball is indeed somewhere; it is just not where it is supposed to be. The apparent contradiction lies in one thinking that an infinite amount of tasks can be performed in a finite amount of time. However, since the ball is only in one place we have a firm proof of Black's argument. His infinity machines, which do an infinite number of tasks within a finite amount of time cannot work and, therefore, show one proof an infinite amount of tasks cannot be undertaken in a finite amount of time.

What are the consequences of Max Black being proven right? If he is right, then space is not infinitely divisible. If space were infinitely divisible then one would be forced to traverse an infinite amount of points in space in order to move anywhere. This concept of space being infinitely divisible, says Black, is due to looking at the world with a mathematical abstraction. Black does not deny that space is mathematically infinitely divisible. However, he says that this mathematical concept is not accurate for describing the real world.

Finally we can come to the solution proposed by Black. He stipulates that the point Zeno was trying to make is not that motion is impossible. He knew that people moved. Zeno was actually trying to show the fallibility of relying purely upon mathematical views of the world. Therefore, Black's solution is that the runner can complete his journey because he doesn't have an infinite amount of them to do. There is a finite amount of space between start and finish and the runner can go through all of this space.